CURRICULUM ESSENTIALS

MATHEMATICS – Grade 4

Unit 3

School Year 2011-12

Unit Names and Pacing

Mathematics — Grade 4			
		Pacing	
Unit Number	Name of Unit	Calendar Dates	Number of Weeks
Unit 1	Working With Numbers	08/31/11 – 10/05/11	5
Buffer days - 4			
Unit 2	Extending the Number System	10/12/11 – 11/18/11	5
Buffer days - 5			
Unit 3	Patterns	12/05/11 – 01/11/12	3.5
Buffer days - 4			
Unit 4	Representing Data	01/23/12 – 02/10/12	3
Buffer days - 5			
Unit 5	Classifying Shapes	02/27/12 – 03/16/12	3
Buffer days - 4			
Unit 6	Measurements	03/26/12 – 05/04/12	5
Buffer days - 5			
Unit 7	Personal Financial Literacy	05/14/12 – 06/04/12	3
Buffer days - 2			

Priority Standards by Unit

Mathematics — Grade 4								
Priority Standards		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
Standard 1 – Number Sense, Properties, and Operations								
MA.4.1.2.a:	Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF) *(See Standards MA.4.1.2.a.ii. and MA.4.1.2.a.iii. following here, for continuance.)*		x	x				
MA.4.1.2.a.ii:	Use the principle of fraction equivalence to recognize and generate equivalent fractions. (CCSS: 4.NF.1)		x					x
MA.4.1.2.a.iii:	Compare two fractions with different numerators and different denominators, and justify the conclusions. (CCSS: 4.NF.2)		x					
MA.4.1.2.b:	Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF)		x	x				
MA.4.1.3.a:	Use place value understanding and properties of operations to perform multi-digit arithmetic. (CCSS: 4.NBT)	x	x					x
MA.4.1.3.a.i:	Fluently add and subtract multi-digit whole numbers using standard algorithms. (CCSS: 4.NBT.4)	x					x	x
MA.4.1.3.a.ii:	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. (CCSS: 4.NBT.5)	x						
MA.4.1.3.a.iii:	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 4.NBT.6)	x						
MA.4.1.3.b:	Use the four operations with whole numbers to solve problems. (CCSS: 4.OA)	x						x
MA.4.1.3.b.iii:	Multiply or divide to solve word problems involving multiplicative comparison. (CCSS: 4.OA.2)	x		x				
MA.4.1.3.b.iv:	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. (CCSS: 4.OA.3)	x	x					

Mathematics — Grade 4								
Priority Standards		Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7
MA.4.1.3.b.v:	Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3)		x					x
MA.4.1.3.b.vi:	Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3)	x		x	x	x		x
Standard 2 – Patterns, Functions, and Algebraic Structures								
MA.4.2.1.a:	Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself. (CCSS: 4.OA.5)		x	x				
MA.4.2.1.a.iv:	Find the unknown in simple equations			x				x
MA.4.2.1.b:	Apply concepts of squares, primes, composites, factors, and multiples to solve problems	x		x				
MA.4.2.1.b.ii:	Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)	x		x				
Standard 3 – Data Analysis, Statistics, and Probability								
MA.4.3.1.b:	Solve problems involving addition and subtraction of fractions by using information presented in line plots. (CCSS: 4.MD.4)		x		x			
Standard 4 – Shape, Dimension, and Geometric Relationships								
MA.4.4.1.a:	Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (CCSS: 4.MD)						x	x
MA.4.4.1.a.i:	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. (CCSS: 4.MD.1)						x	
MA.4.4.1.a.iii:	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. (CCSS: 4.MD.2)						x	x
MA.4.4.1.a.v:	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. (CCSS: 4.MD.3)						x	
MA.4.4.2.b:	Identify points, line segments, angles, and perpendicular and parallel lines in two-dimensional figures. (CCSS: 4.G.1)					x	x	
MA.4.4.2.c:	Classify and identify two-dimensional figures according to attributes of line relationships or angle size. (CCSS: 4.G.2)					x		

Priority Standards

Mathematics — Grade 4

Standard 1 – Number Sense, Properties, and Operations

MA.4.1.2.a:	**Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF) *(See Standards MA.4.1.2.a.ii. and MA.4.1.2.a.iii. following here, for continuance.)***
MA.4.1.2.a.ii:	**Use the principle of fraction equivalence to recognize and generate equivalent fractions. (CCSS: 4.NF.1)**
MA.4.1.2.a.iii:	**Compare two fractions with different numerators and different denominators, and justify the conclusions. (CCSS: 4.NF.2)**
MA.4.1.2.b:	**Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF)**
MA.4.1.3.a:	**Use place value understanding and properties of operations to perform multi-digit arithmetic. (CCSS: 4.NBT)**
MA.4.1.3.a.i:	**Fluently add and subtract multi-digit whole numbers using standard algorithms. (CCSS: 4.NBT.4)**
MA.4.1.3.a.ii:	**Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. (CCSS: 4.NBT.5)**
MA.4.1.3.a.iii:	**Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. (CCSS: 4.NBT.6)**
MA.4.1.3.b:	**Use the four operations with whole numbers to solve problems. (CCSS: 4.OA)**
MA.4.1.3.b.iii:	**Multiply or divide to solve word problems involving multiplicative comparison. (CCSS: 4.OA.2)**
MA.4.1.3.b.iv:	**Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. (CCSS: 4.OA.3)**
MA.4.1.3.b.v:	**Represent multistep word problems with equations using a variable to represent the unknown quantity. (CCSS: 4.OA.3)**
MA.4.1.3.b.vi:	**Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.3)**

Standard 2 – Patterns, Functions, and Algebraic Structures

MA.4.2.1.a:	**Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself. (CCSS: 4.OA.5)**
MA.4.2.1.a.iv:	**Find the unknown in simple equations**
MA.4.2.1.b:	**Apply concepts of squares, primes, composites, factors, and multiples to solve problems**
MA.4.2.1.b.ii:	**Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)**

Standard 3 – Data Analysis, Statistics, and Probability

MA.4.3.1.b:	**Solve problems involving addition and subtraction of fractions by using information presented in line plots. (CCSS: 4.MD.4)**

Unit of Study

Content Area	Math
Grade/Course	4
Unit of Study	Patterns
Unit Number	3
Calendar Dates **Pacing**	12/05/11 – 01/11/12 3.5 weeks (buffer days – 4)
Unit Type	☑ Topical ☐ Skills-based ☐ Thematic

Essential Questions	Corresponding Big Ideas
1. *How can we predict the next element in a pattern?*	1. Patterns follow rules and these rules help us make predictions.
2. *Why do mathematicians use patterns?*	2. Mathematicians use patterns to simplify calculations and to help make predictions about future outcomes.

Priority Standards Supporting Standards

Priority Standards

Standard 1 – Number Sense, Properties, and Operations

MA.4.1.2.a:	**Use ideas of fraction equivalence and ordering to: (CCSS: 4.NF)**
MA.4.1.2.b:	**Build fractions from unit fractions by applying understandings of operations on whole numbers. (CCSS: 4.NF)**
MA.4.1.3.b.iii:	**Multiply or divide to solve word problems involving multiplicative comparison.[1] (CCSS: 4.OA.2)**
MA.4.1.3.vi:	**Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA)**

Standard 2 – Patterns, Functions, and Algebraic Structures

MA.4.2.1.a:	**Generate and analyze patterns and identify apparent features of the pattern that were not explicit in the rule itself.[2] (CCSS: 4.OA.5)**
MA.4.2.1.a.iv:	**Find the unknown in simple equations**
MA.4.2.1.b:	**Apply concepts of squares, primes, composites, factors, and multiples to solve problems**
MA.4.2.1.b.ii:	**Recognize that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)**

[1] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS: 4.OA.2)

[2] For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way. (CCSS: 4.OA.5)

Supporting Standards

Standard 1 – Number Sense, Properties, and Operations

MA.4.1.1.a.i:	Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1)
MA.4.1.1.a.iv:	Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3)
MA.1.1.b.i:	Interpret a multiplication equation as a comparison.[3] (CCSS: 4.OA.1)

Standard 2 – Patterns, Functions, and Algebraic Structures

MA.4.2.1.a.i:	Use number relationships to find the missing number in a sequence
MA.4.2.1.a.ii:	Use a symbol to represent and find an unknown quantity in a problem situation
MA.4.2.1.a.iii:	Complete input/output tables

Standard 3 – Data Analysis, Statistics, and Probability

MA.4.3.1.a:	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). (CCSS: 4.MD.4)

"Unwrapped" Priority Standards Supporting Standards

Priority Standards

Standard 1 – Number Sense, Properties, and Operations

MA.4.1.2.a:	**USE ideas of fraction equivalence and ordering to: (CCSS: 4.NF)**
MA.4.1.2.b:	**BUILD fractions from unit fractions by APPLYING understandings of operations on whole numbers. (CCSS: 4.NF)**
MA.4.1.3.b.iii:	**MULTIPLY or DIVIDE to solve word problems involving multiplicative comparison.[4] (CCSS: 4.OA.2)**
MA.4.1.3.b.vi:	**ASSESS the reasonableness of answers using mental computation and estimation strategies including rounding. (CCSS: 4.OA.**

Standard 2 – Patterns, Functions, and Algebraic Structures

MA.4.2.1.a:	**GENERATE and ANALYZE patterns and identify apparent features of the pattern that were not explicit in the rule itself.(CCSS: 4.OA.5)**
MA.4.2.1.a.iv:	**FIND the unknown in simple equations**
MA.4.2.1.b:	**APPLY concepts of squares, primes, composites, factors, and multiples to solve problems**
MA.4.2.1.b.ii:	**RECOGNIZE that a whole number is a multiple of each of its factors. (CCSS: 4.OA.4)**

[3] e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. (CCSS: 4.OA.1)

[4] e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison. (CCSS: 4.OA.2)

Supporting Standards

Standard 1 – Number Sense, Properties, and Operations

MA.4.1.1.a.i: Explain that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. (CCSS: 4.NBT.1)

MA.4.1.1.a.iv: Use place value understanding to round multi-digit whole numbers to any place. (CCSS: 4.NBT.3)

MA.1.1.b.i: Interpret a multiplication equation as a comparison.[5] (CCSS: 4.OA.1)

Standard 2 – Patterns, Functions, and Algebraic Structures

MA.4.2.1.a.i: Use number relationships to find the missing number in a sequence

MA.4.2.1.a.ii: Use a symbol to represent and find an unknown quantity in a problem situation

MA.4.2.1.a.iii: Complete input/output tables

Standard 3 – Data Analysis, Statistics, and Probability

MA.4.3.1.a: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). (CCSS: 4.MD.4)

[5] e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. (CCSS: 4.OA.1)

"Unwrapped" Concepts (students need to know)	**"Unwrapped" Skills** (students need to be able to do)	**Bloom's Taxonomy Levels**
• fractions ○ equivalence ○ ordering ○ unit fractions ○ operations ○ whole numbers • multiplication and division ○ word problems ○ multiplicative comparison • reasonableness of answers ○ mental computation ○ estimation strategies ○ rounding • patterns ○ features ○ rules • simple equations ○ unknowns • concepts ○ squares ○ primes ○ composites ○ factors ○ multiples • whole number ○ multiples of factors	• USE (fraction equivalence and ordering) • BUILD (fractions from unit fractions) by APPLYING (understandings of operations on whole numbers) • MULTIPLY or DIVIDE (to solve word problems involving multiplicative comparison) • ASSESS (the reasonableness of answers using mental computation and estimation strategies including rounding) • GENERATE and ANALYZE (and identify apparent features of the pattern that were not explicit in the rule itself) • FIND (the unknown in simple equations) • APPLY (concepts of squares, primes, composites, factors, and multiples to solve problems) • RECOGNIZE (that a whole number is a multiple of each of its factors)	3 3 3 4 4 3 3 3

Essential Questions	**Corresponding Big Ideas**
1. *How can we predict the next element in a pattern?*	1. Patterns follow rules and these rules help us make predictions.
2. *Why do mathematicians use patterns?*	2. Mathematicians use patterns to simplify calculations and to help make predictions about future outcomes.

Note: As assessments were developed, items were designed to reflect the format, vocabulary used, and frequency of items on state assessments.

SECTION 1
Selected-Response
(Multiple-Choice, Matching, True-False, Select from Provided Word List)

> ***Note to Teacher:*** The following selected-response questions were developed by a team of teachers. Details for you to notice include:
>
> 1. **All** of the "unwrapped" skills, related concepts, and matching Bloom levels are listed.
> 2. Those in **bold** are assessed through the selected-response format.
> 3. Each assessment item **directly matches** the approximate level of rigor for each skill.
> 4. **Distracters** (incorrect answer choices) are plausible and/or reflective of common student errors.

(3) USE (fraction equivalence and ordering)
(3) BUILD (fractions from unit fractions) by APPLYING (understandings of operations on whole numbers)
(3) MULTIPLY or DIVIDE (to solve word problems involving multiplicative comparison)
(4) ASSESS (the reasonableness of answers using mental computation and estimation strategies including rounding)
(4) GENERATE and ANALYZE (and identify apparent features of the pattern that were not explicit in the rule itself)
(3) FIND (the unknown in simple equations)
(3) APPLY (concepts of squares, primes, composites, factors, and multiples to solve problems)
(3) RECOGNIZE (that a whole number is a multiple of each of its factors)

Student Directions: Choose the correct answer for questions 1 – 6.

1. What does y equal in the following equation? $y = 14 \div 2$ (MA.4.2.1.a.iv) TC

   A. 16
   B. 7
   C. 8
   D. 12

SECTION 1 (cont.)
Selected-Response
(Multiple-Choice, Matching, True-False, Select from Provided Word List)

2. There are 4 beans in a jar. Each day 3 beans are added. How many beans are in the jar on Day 3? (MA.4.2.1.a) TC

DAY	OPERATION	BEANS
0	3 x 0 + 4	4
1	3 x 1 + 4	7
2	3 x 2 + 4	10
3	3 x 3 + 4	?

A. 10
B. 13
C. 15
D. 12

3. A chef opened a jar of olives. He used them to make 2 identical plates. There were 37 olives on each plate. Which equation, when solved, will tell how many olives were in the jar? In the equation, *n* = the number of olives. (MA.4.2.1.a.iv) www.ixl.com

A. $n \div 2 = 37$
B. $n \div 2 = 74 \div 37$
C. $2 \div n = 37$
D. $2 - 37 = n$

4. A new video is becoming popular. The local video store ordered 1 copy in May, 4 copies in June, 16 copies in July, and 64 copies in August. If this pattern continues, how many copies will the video store order in September? (MA.4.2.1.a) TC

A. 122
B. 830
C. 256
D. 563

5. Gina has *g* gumballs. Mandy has 92 fewer gumballs than Gina. Choose the expression that shows how many gumballs Mandy has. (MA.4.2.1.a.iv) www.ixl.com

A. $g - 92$
B. g
C. $g + 92$
D. 92

SECTION 1 (cont.)
Selected-Response
(Multiple-Choice, Matching, True-False, Select from Provided Word List)

6. What is the rule for the in/out box? (MA.4.2.1.a) TC

IN	OUT
20	60
130	170
70	110
340	380

Rule: ______?

A. add 40
B. add 30
C. add 20
D. add 45

Answer Key

1. B
2. B
3. A
4. C
5. A
6. A

SECTION 2
Constructed-Response
(Short Answer)

Note to Teacher: The following short constructed-response questions were developed by a team of teachers. Details for you to notice include:

1. **All** of the "unwrapped" skills, related concepts, and matching Bloom levels are listed.
2. Those in **bold** are assessed through the constructed-response format.
3. Each assessment item **directly matches** the approximate level of rigor for each skill.

(3) USE (fraction equivalence and ordering)
(3) BUILD (fractions from unit fractions) by APPLYING (understandings of operations on whole numbers)
(3) MULTIPLY or DIVIDE (to solve word problems involving multiplicative comparison)
(4) ASSESS (the reasonableness of answers using mental computation and estimation strategies including rounding)
(4) GENERATE and ANALYZE (and identify apparent features of the pattern that were not explicit in the rule itself)
(3) FIND (the unknown in simple equations)
(3) APPLY (concepts of squares, primes, composites, factors, and multiples to solve problems)
(3) RECOGNIZE (that a whole number is a multiple of each of its factors)

SECTION 2 (cont.)
Constructed-Response
(Short Answer)

Student Directions: Respond to the following question. Show your work. Your responses will be evaluated using the following Short Answer Scoring Guide.

7. What do the following numbers have in common? How do you know? (MA.4.2.1.b) TC

2, 5, 7, 11, 13 and 17

The numbers listed all: ____.

I know this because: ____

____.

An array of one of these numbers would look like this:

 # 7 – Short Answer Scoring Guide

Proficient (3)

- Correctly identifies what the list of numbers has in common
- Explains how they know what the numbers have in common
- Provides an accurate array

Partially Proficient (2)

- Meets 2 of the "Proficient" criteria

Unsatisfactory (1)

- Meets 1 or fewer of the "Proficient" criteria

No Response (0)

Note to Teacher

Answers and Other Information

All of the numbers are prime. You know this because they all have factors that are only 1 and themselves. The array can be any of the numbers x 1.

SECTION 2 (cont.)
Constructed-Response
(Short Answer)

Student Directions: Respond to question 8. Show your work. Explain how you know that the answer is correct. Your responses will be evaluated using the following Short Answer Scoring Guide.

8. Is 5,000 a reasonable estimate of 54 x 1200? Why or why not? Use your understanding of estimation strategies to explain your answer. (MA.4.1.3.vi) TC

 # 8 – Short Answer Scoring Guide

Advanced (4)

- **All "Proficient" criteria plus:**
  - Makes real-life connection to home or school

Proficient (3)

- Correctly explains why or why not 5,000 is or isn't a reasonable estimate
- Uses estimation strategies to explain answer

Partially Proficient (2)

- Meets 1 of the "Proficient" criteria

Unsatisfactory (1)

- Meets none of the "Proficient" criteria

No Response (0)

Note to Teacher

Possible Solutions:

It is not a reasonable estimate because –

50 x 1000 = 50,0000

OR

5 x 1200 = 6,000

50 x 1200 = 60,000

SECTION 2 (cont.)
Constructed-Response
(Short Answer)

Student Directions: Respond to the following question. Your responses will be evaluated using the following Short Answer Scoring Guide.

9. A red hat costs $18. A blue hat costs $6. Use multiplicative comparison to show how many times as much does the red hat costs compared to the blue hat? Write an equation to support your answer. Use the relationship between multiplication and division to explain how you know that the answer is correct. Show your work. (MA.4.1.3.b.iii) TC

   The red had costs _________times more than the blue hat.

 # 9 – Short Answer Scoring Guide

Proficient (3)

- Accurately calculates the multiplicative comparison
- Creates an equation to support how the answer was derived
- Uses the relationship between multiplication and division to explain how you know that the answer is correct

Partially Proficient (2)

- Meets 2 of the "Proficient" criteria

Unsatisfactory (1)

- Meets 1 or fewer of the "Proficient" criteria

No Response (0)

Note to Teacher
Possible solutions:
18 ÷ 6 = p
6 x p = 18

SECTION 2 (cont.)
Constructed-Response
(Extended-Response)

Note to Teacher: The following extended constructed-response items were developed by a team of teachers. Details for you to notice include:

1. **All** of the "unwrapped" skills, related concepts, and matching Bloom levels are listed.
2. Those in **bold** are assessed through the constructed-response format.
3. Each assessment item **directly matches** the approximate level of rigor for each skill.

(3) USE (fraction equivalence and ordering)
(3) BUILD (fractions from unit fractions) by APPLYING (understandings of operations on whole numbers)
(3) MULTIPLY or DIVIDE (to solve word problems involving multiplicative comparison)
(4) ASSESS (the reasonableness of answers using mental computation and estimation strategies including rounding)
(4) GENERATE and ANALYZE (and identify apparent features of the pattern that were not explicit in the rule itself)
(3) FIND (the unknown in simple equations)
(3) APPLY (concepts of squares, primes, composites, factors, and multiples to solve problems)
(3) RECOGNIZE (that a whole number is a multiple of each of its factors)

10. Student Directions: Answer parts A, B and C. Use mathematical vocabulary to explain your answers. Show your work and use the correct units where appropriate. Your responses will be evaluated using the following Extended-Response Scoring Guide. TC

Part A: How does knowing that $\frac{3}{4}$ is closer to 1 than $\frac{1}{2}$ is, allow you to order fractions?

Part B: Amanda grew two plants for the science fair. The first plant was twelve centimeters tall. The second plant was $\frac{1}{2}$ as tall. How tall was the second plant?

Part C: If the third plant was ¾ as tall as the first would it be taller or shorter than the plant that is $\frac{1}{2}$ as tall? How do you know?

SECTION 2 (cont.)
Constructed-Response
 # 10 – Extended-Response Scoring Guide

Advanced (4)

- **All "Proficient" criteria plus**
  - Makes real-life connection to home or school

Proficient (3)

- Answers all parts of the question
- Correctly solves parts B and C
- Uses mathematical vocabulary to explain their answers
- Shows work
- Uses correct label
- Explains how answer in part C is known

Partially Proficient (2)

- Meets 4 or 5 of the "Proficient" criteria

Unsatisfactory (1)

- Meets 3 or fewer of the "Proficient" criteria

No Response (0)

Note to Teacher

Possible answers:

A. Being able to compare a number to unit fractions helps us order them and know which one is bigger.
B. The second plant is 6 cm.
C. The third plant would be 9 cm, which is bigger than the second plant. I know this because $\frac{3}{4}$ is closer to one so it needs to be bigger. I also know this because $\frac{3}{4}$ x 12 = 9.

SECTION 3
Big Idea Responses to
Essential Questions

Student Directions: Write a Big Idea response for **each** of the following Essential Questions. Include vocabulary terms you have learned. Your responses will be evaluated using the following Big Ideas Scoring Guide.

1. How can we predict the next element in a pattern?

2. Why do mathematicians use patterns?

Big Ideas Scoring Guide

Advanced (4)

- **All "Proficient" criteria plus**
   - Provides example(s) as part of responses
   - Makes connections to other areas of school or life

Proficient (3)

- States all Big Ideas correctly in own words
- Includes vocabulary terms in responses

Partially Proficient (2)

- Meets 1 of the "Proficient" criteria

Unsatisfactory (1)

- Not yet able to respond correctly
- Comments:

No Response (0)

SECTION 3
Big Idea Responses to
Essential Questions

Student Directions: Write a Big Idea response for **each** of the following Essential Questions. Include vocabulary terms you have learned. Your responses will be evaluated using the following Big Ideas Scoring Guide.

1. How can we predict the next element in a pattern?

2. Why do mathematicians use patterns?

Big Ideas Scoring Guide

Advanced
- **All "Proficient" criteria plus**
  - Provides example(s) as part of responses
  - Makes connections to other areas of school or life

Proficient
- States all Big Ideas correctly in own words
- Includes vocabulary terms in responses

Partially Proficient
- Meets 1 of the "Proficient" criteria

Unsatisfactory
- Not yet able to respond correctly
- Comments:

Subject: Mathematics
Grade: 4
Unit of Study: Patterns

Big Idea: Patterns follow rules and these rules allow us to make predictions.

Patterns with Fractions
- equivalence
- ordering
- building fractions from unit fractions

Assessment Opportunity
Possible assessments: Exit Tickets and “Make A Whole Game”

Patterns with Numbers
- mental computation
- estimation
- multiplicative comparison
- square numbers
- prime numbers
- composite numbers
- factors

Assessment Opportunity
Possible assessments: Number Talks to assess mental math.

Simple Equations
- unknown variables
- generate terms

Assessment Opportu
Possible

Big Idea: Mathematicians use patterns to simplify calculations and to help make

Learning Targets: Students will understand that mathematical patterns help us solve real word problems.

Criteria for Success: Students will score proficiency on the post-CFA.

CHAPTER TWELVE

HOW MUCH IS BLUE?

A PATTERN BLOCK ACTIVITY

Overview

In this activity, students are shown a design made with thirteen pattern block pieces—three green triangles, six blue parallelograms, three red trapezoids, and one yellow hexagon. The problem they solve is: "What fractional part of this design is blue?" The challenge of this problem for students is that they must think about both the fractional parts of the total design and the areas of the individual pattern block pieces. Along with finding the answer, students have to explain their reasoning.

Materials

- pattern blocks, enough so that each child can build the design and have extra blocks as well, about 1 bucket per 6 students
- *Pattern Block Design* worksheet, 1 per student (see Blackline Masters)
- optional for extension: *Pattern Block Triangle Paper* worksheet, 1 per student (see Blackline Masters)

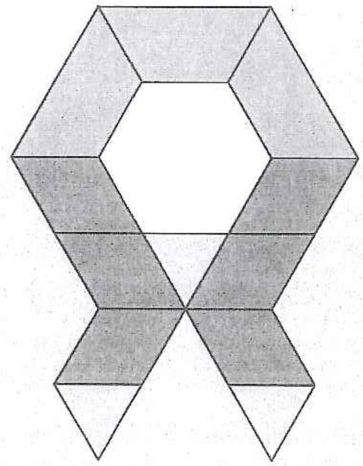
Time

- two class periods, plus extra time for students to solve one another's puzzles

Teaching Directions

1. On chart paper or on an overhead transparency, draw the design as shown (see next page) so all students can see it.

2. Discuss with the students what they know about pattern blocks. Then write on the board the problem the students are to solve: *What fractional part of this design is blue?* Ask the children to think about the problem quietly by themselves before you begin a class discussion.



3. After a few moments, begin a class discussion by asking: "Who has an estimate of what the answer might be?" Discuss their ideas.

4. Then give each student a copy of the design drawn to the actual size of pattern blocks. (See Blackline Masters.) Also, distribute pattern blocks. Give the class directions: "Prepare your papers by writing the question at the top. Figure out the answer by using the blocks, drawing pictures, or doing whatever will help you. Then write about how you reasoned." If you have students work in pairs, it's still a good idea for each to write an individual paper.

5. Circulate as the students work, giving help as needed.

6. The next day, have students who are willing report how they solved the problem. To extend the experience, give the following directions: "Each of you should make your own design using an assortment of green, blue, red, and yellow pattern blocks. Draw your design on pattern block paper and then pose a question: What fractional part of the design is ________? (Choose one of the colors.) On a separate sheet, record the answer and explain your reasoning."

7. Check the students' papers for accuracy. Then either devote class time for all students to figure out answers to one another's designs or organize the designs and answers into two folders and use them for a choice activity.

Teaching Notes

In the November 1995 issue of the NCTM journal *Teaching Children Mathematics* (Volume 2, Number 3), Janet H. Caldwell, professor of mathematics at Rowan College in New Jersey, wrote an article titled "Communicating About Fractions with Pattern Blocks." After reading the article, I tried the activity and found it effective for engaging students and assessing their understanding.

An interesting aspect of the problem is that while six of the thirteen blocks are blue, the blocks of different colors are each different sizes. Therefore, it's not true that six-thirteenths of the design is blue. In order to solve this problem correctly, students have to consider the area of each of the pieces.

For this activity, the students should have access to pattern blocks. As with all manipulative materials, it's important that students are comfortable and familiar with the blocks before being asked to focus on this problem. They should have had prior experience exploring pattern blocks and the relationships among them. If this is not the case, then I recommend first trying the introductory activities presented in Chapter 4, "Exploring Fractions with Pattern Blocks."

The Lesson

DAY 1

To prepare for the lesson, I drew on chart paper an enlarged version of the design made from pattern blocks and posted it for all of the students to see.

"It's a rocket!" Sam said.

"Maybe it's a kite," Michael added.

I didn't comment on the boys' remarks but addressed the class. "What do you know about pattern blocks?" I asked. The students were familiar with pattern blocks and I wanted to focus their attention on them before presenting the problem.

"Two greens make a blue," Emma said.

"And six greens make a yellow," Nick added.

"You can make a yellow in lots of different ways," Sean said.

"Not so many ways," Amy responded. "Just six, I think. Or maybe seven." Earlier we had investigated the different combinations of red, blue, and green pattern blocks that could be used to build the yellow hexagon. Then, considering the hexagon as the whole, we assigned fractional values to each of the other pieces. (See the *Exploring Fractions with Pattern Blocks* lesson, pages 39–45.)

"I know the names of the shapes," Alma said. She then correctly recited them, "Triangle, parallelogram, hexagon, and trapezoid."

"I remember that you can use a red one and three triangles to make the yellow hexagon," Ramon said.

Some hands were still raised, but I said, "You seem to know about the pattern blocks, so rather than hearing more ideas, let me tell you about the problem I'd like you to solve."

I directed their attention to the chart paper. "The problem is about this design. I'll write it on the board." Writing the problem as well as giving it orally is helpful for children who don't always absorb what they hear. I read as I wrote:

What fractional part of this design is blue?

"First think about this quietly by yourself," I said. "Then we'll have a class discussion about your ideas."

The room got quiet. After a few moments, I asked the class, "Who has an estimate of what the answer might be?" About seven or eight students raised their hands immediately. I waited, and several more students raised their hands. I called on Sarah.

"I think it's a little less than half," Sarah said.

"Why do you think that?" I asked.

Sarah replied, "See, first I tried to count the blocks, but it was hard to do from here. So then I just looked at it and it looks like it's less than half." There were some murmurs of agreement from the class.

"Any other ideas?" I asked. I called on Paul.

"I counted, and I think it's six-thirteenths," Paul said.

"What did you count?" I asked.

"The blocks," Paul answered. "There are thirteen blocks and six of them are blue, so I would write six over thirteen."

Sophie chimed in, "I don't think that can be." She turned to me and asked, "Can I explain?" I nodded.

Sophie then turned to Paul. "You can't do that, Paul," she said. "You can't count all the blocks as the same because they're different sizes so they're not equal. The sizes have to be equal in fractions."

Paul thought for a moment and then said, "Oh yeah." He didn't seem convinced, however.

"Do you have an idea about an answer?" I asked Sophie.

"I'm not sure yet," she answered. "But I started figuring it out. I think it would be easier to figure with the blocks. Then I could cover the design in greens." Several children agreed.

"I was going to cover it in blues," Daniel added.

"That's what I was going to do," Sean said.

"Me, too," several others added.

"Could we try it with the blocks?" Amy asked.

The children were getting excited. I quieted them by asking for their attention and then explaining what they were to do. "I'll give each of you a copy of the design drawn to the actual size of pattern blocks. Also, I'll give a bag of pattern blocks to each group. While I do this, you can get your papers ready by each writing at the top of your paper your name, the date, and the question. Then you can figure out the answer by using the blocks, drawing pictures, or doing whatever will help you. Finally, you need to write about how you reasoned."

"Can we work together?" Tina asked.

"You can talk about your ideas with one another," I said. "But you each should write your own paper. Remember, it's fine to use an idea you get from someone else, as long as it makes sense to you and you can explain it with your own words." I distributed the pattern blocks as the students organized their papers.

Observing the Students

I circulated as the students worked, answering questions, prodding for more information, and refocusing students when necessary.

Paul asked me, "Should I write about what I thought first and then give another idea?"

"That would be good," I said. "Then I'll have a record of how you changed your thinking."

After first writing about the idea he had presented to the class, Paul covered the design with green triangles. He then added to his paper: *there were 30 triangles. then, I thought, "If I added 2 triangles together, and there are 6 diamonds, then I will just divid 30 in half which is 15." So $\frac{6}{15}$ is blue.* (See Figure 12–1.)

Sophie worked quickly, doing what she had said she would—covering the design

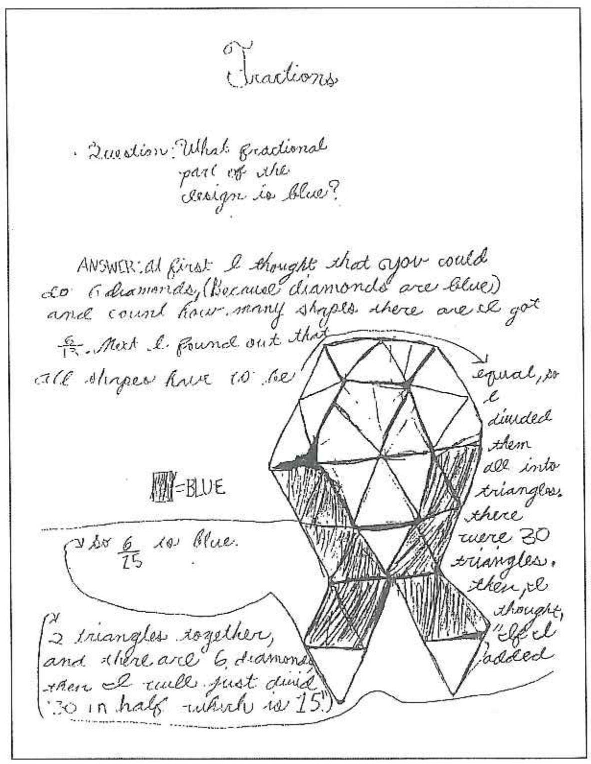


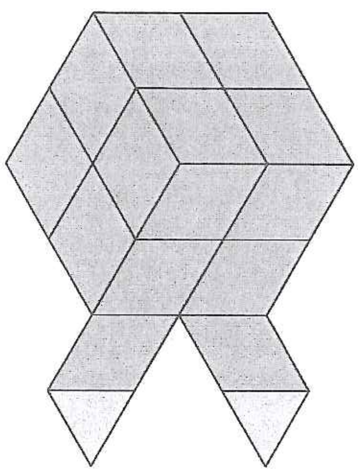
Figure 12–1 *Paul described his original idea and then his revised idea.*

with green triangles. She counted them to find that thirty triangles covered the design. Sophie wrote: *I think that it could be* $\frac{12}{30}$ *because if you divide all the peices into triangles there are 30, and there are twelve triangles for 6 blues.* $\frac{12}{30} = \frac{6}{15}$.

Daniel and Sean worked together to cover the design with blue parallelograms. "It doesn't work exactly," Daniel said.

"What do you mean?" I asked.

"Look," Sean explained. "You can't really cover it. You'd have to cut a blue one in half to make it really fit. But we used triangles." The boys had discovered that the shape of the design makes it impossible to cover it entirely with blue parallelograms. They had used fourteen blue parallelograms and two green triangles.

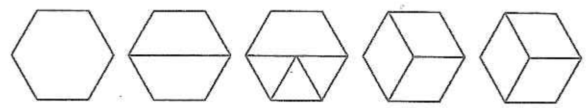


"What are you going to do now?" I asked.

"No problem," Daniel said. "It's the same as fifteen blues so the answer is six-fifteenths. We're done."

"So you're ready to write now," I said as a reminder. The boys nodded and reached for their papers.

Alma and Amy worked together and came up with an answer of two-fifths. They had rearranged the thirteen blocks in the design into five hexagons, each the same size as the yellow pattern block hexagon. The six blue blocks formed two of the hexagons.



Alma had written:

We maid the blocs so that there were 5 hexegons, and 2 of the hexegons were blue and that is all the blues. Answer $\frac{2}{5}$ *is blue.*

The five hexagons were on her desk. (See Figure 12–2.)

Because Sophie had finished her paper by then, I told her that Alma and Amy thought about the problem in a completely different way and figured out that the design was two-fifths blue.

"Are they right?" Sophie asked.

"Go take a look at what they did and see if you agree," I answered. Emma went with Sophie, and in a moment I heard their comments of admiration. The girls returned to

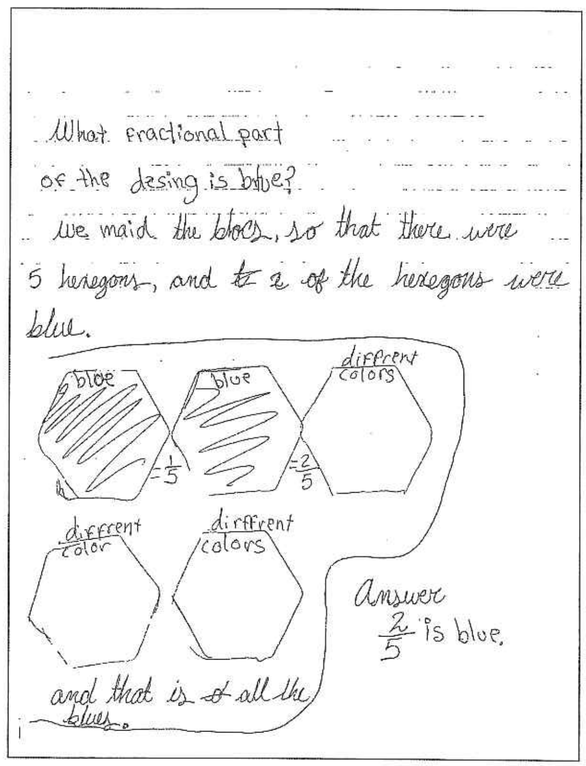


Figure 12–2 *Alma and Amy solved the problem by rearranging the 13 blocks into 5 hexagons, using 6 blue blocks to form 2 of the hexagons.*

their desks to rearrange their blocks into hexagons.

Katy brought her paper to me with a sly smile. I read what she wrote:

> *I say a little bit more than a third of the drawing is blue. I know this because I divided the whole drawing into diamonds. After I did that I counted the diamonds and got 15 and know that $\frac{1}{3}$ of 15 is 5 and 6 is only 1 number greater than 5.*

I laughed. I enjoy when children think about something in a way I never had imagined. "Your reasoning is absolutely correct," I told Katy. "Could you also figure out what is the exact fraction that represents the blue part of the design?"

"Isn't my answer good enough?" Katy challenged me.

I answered, "It could be, depending on what the need was for figuring this out, but I'm also interested in learning as much as I can about what you know about fractions. I'm curious if you can figure out a precise answer."

Katy wasn't happy with my request, but she returned to her desk and added: *However, the exact fraction of blue diamonds is $\frac{6}{15}$.* (See Figure 12–3.)

I checked on Joey and Sarah. Together they had covered the duplicated design with green triangles as Sophie had. Joey watched as Sarah counted the blocks.

"There are thirty," she said to Joey. "And that would be fifteen with the blue diamonds." Sarah started covering the green triangles with blue blocks, and Joey joined in when he realized what she was doing.

"So six out of the fifteen are blue," Sarah said. "That's it." She turned to write. At the top of her paper, she wrote *$\frac{6}{15}$ths.* Then she explained: *We took the picture and devided all the shapes into triangles and there were 30 and as you now 2 triangles is the same as a diamond.* She drew thirty triangles and drew circles to group them into twos to show there would be fifteen diamonds.

Joey watched for a moment and then started to write: *We took the picter and we cut them all into tiangles. We got 30 tiangels and drew them and sircled tow of them and counted the sicals* [circles] *and got 15.* He turned to Sarah for help about what to do next.

Just then, the resource teacher came for Joey. (Joey went to the resource room for extra help twice a week.) She stopped and looked at what Joey had written. "Can you take that with you, Joey?" she asked. "I'd like you to tell me about the problem you're working on. It looks interesting."

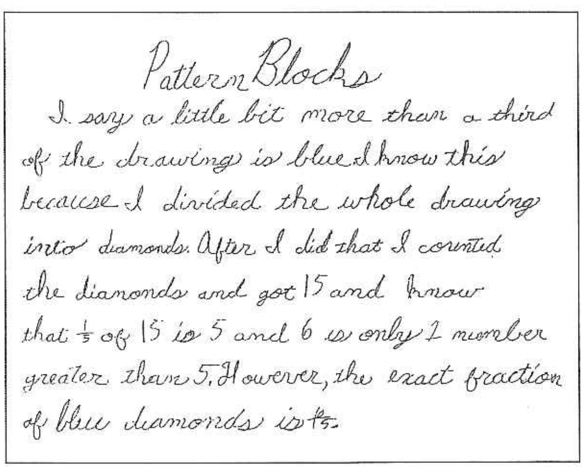
"Can I?" Joey asked me.

"Sure," I said. "And take a bag of pattern blocks, too."

Eli is a dreamy boy who enjoys drawing and writing poems and stories. He knows the words to many of the rock songs played on the radio and likes to write down the lyrics and discuss what they mean. On this day, Eli had trouble settling into the problem, and I stopped by his desk several times to help him refocus. Once Eli focused, however, he wouldn't leave for recess because he hadn't finished writing about his idea.

He wrote:

> *I believe the fraction is $\frac{1}{3}$ and a $\frac{1}{15}$. The reason I think this is because I did the work on paper. First I counted all the blues, I counted 6. Then I used only blues to make the whole*


Pattern Blocks

I say a little bit more than a third of the drawing is blue I know this because I divided the whole drawing into diamonds. After I did that I counted the dianonds and got 15 and know that $\frac{1}{3}$ of 15 is 5 and 6 is only 1 number greater than 5. However, the exact fraction of blue diamonds is $\frac{6}{15}$.

Figure 12–3 *Katy initially estimated the answer but then revised it to show the exact answer.*

thing that time I counted 15 blues. I finally figured out it was $6\frac{1}{15}$s, but that wasn't enough.

Eli then drew a circle and divided it into fifteen segments. He marked six of the segments and wrote next to the circle: *$\frac{1}{3}$ and a $\frac{1}{15}$*. (See Figure 12–4.)

"I'm done," he said, now eager to go outside.

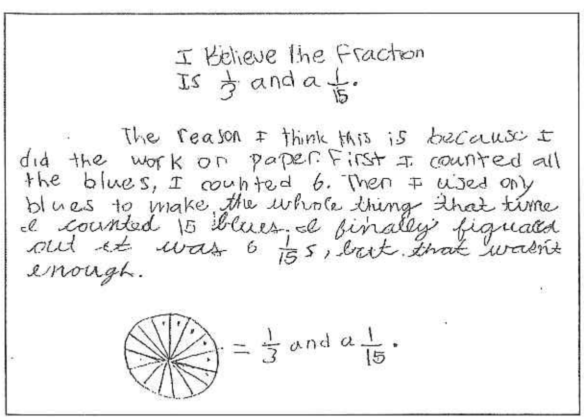
"Wait just one minute," I said. "Help me understand your idea."

Impatiently, but with confidence, Eli said, "Well, five-fifteenths makes a third, okay? But there's one piece left. And that's another fifteenth. So it's one-third and one-fifteenth. Okay?" I nodded and Eli raced to the door.

DAY 2

I began class by having students volunteer to make presentations about their ideas. It was valuable, I thought, for students to hear other ways of thinking about the problem. After the presentations, I gave directions to the class.

I said, "Today you'll each make a pattern block design of your own. I have two guidelines. One is that you must use only green, blue, red, and yellow blocks, as were used in the design I showed you yesterday. The other guideline is that your design must fit on pattern block paper so you can reproduce it. It can't be larger than that." I held up a piece of the pattern block paper as a reminder; students were familiar with it from prior activities.


I Believe the Fraction Is $\frac{1}{3}$ and a $\frac{1}{15}$.

The reason I think this is because I did the work on paper First I counted all the blues, I counted 6. Then I used only blues to make the whole thing that time I counted 15 blues. I finaly figuared out it was $6\frac{1}{15}$s, but that wasn't enough.

$= \frac{1}{3}$ and a $\frac{1}{15}$.

Figure 12–4 *Eli explains in his paper that $\frac{6}{15}$ is the same as $\frac{1}{3}$ and $\frac{1}{15}$.*

I continued with directions, "When you have recorded your design, write a problem at the top of the paper." I wrote on the board:

What fractional part of the design is _______?
(Choose one of the colors.)

"Do we have to use all four colors?" Daniel asked.

I answered, "Not necessarily."

"Can we use the orange squares and tan diamonds?" Sam wanted to know.

"No, you can't," Sophie piped up. "They don't work on the pattern block paper."

"Oh yeah," Sam remembered. Not only do the blocks not fit on the pattern block paper, but figuring their areas would be a challenge beyond the students' ability.

I gave the final instructions. "On a separate sheet, write the problem you posed, then record the answer and explain your reasoning. Before you turn in your papers, have someone try your problem to check that your answer is correct. When you check someone's paper, write your name at the bottom so that I know that the paper has been checked. Also, be sure to put your name on both your design and your answer paper." The directions seemed clear to the students and they got to work.

That night I checked their papers for accuracy. All but three were correct and I made time to talk with those students over the next two days, both the students who had created the designs and the students who had signed the papers. After all of the errors had been resolved, I put their designs into one folder and their answers in another. I showed the folders to the class.

"The problems are available as another choice activity," I explained. "You can pick any design, solve the problem, and then find the answer in the other folder. If you can't figure out the answer, or you get an answer that's different, then talk with either the person who created the design or the person who signed the answer sheet." I showed the class where I was putting the folders and added "Pattern Block Problems" to the choice list.

Questions and Discussion

▲ ***Why did you draw the design on a chart to begin the lesson instead of giving the students their own copies?***

I know posting the design made it difficult for the students to examine it and count the individual blocks, but I wanted to avoid having students begin work individually on the problem before we had had a class discussion.

▲ ***Why did you ask students to think quietly first before the discussion?***

While I generally allow and encourage talking in class, there are times when I think it's valuable for students to focus on a problem by themselves. This gives them a chance to collect their thoughts. I've learned that when I teach complex topics like fractions, students may understand what others say but have difficulty explaining their thinking on their own. They need ample opportunities to formulate their own ideas.

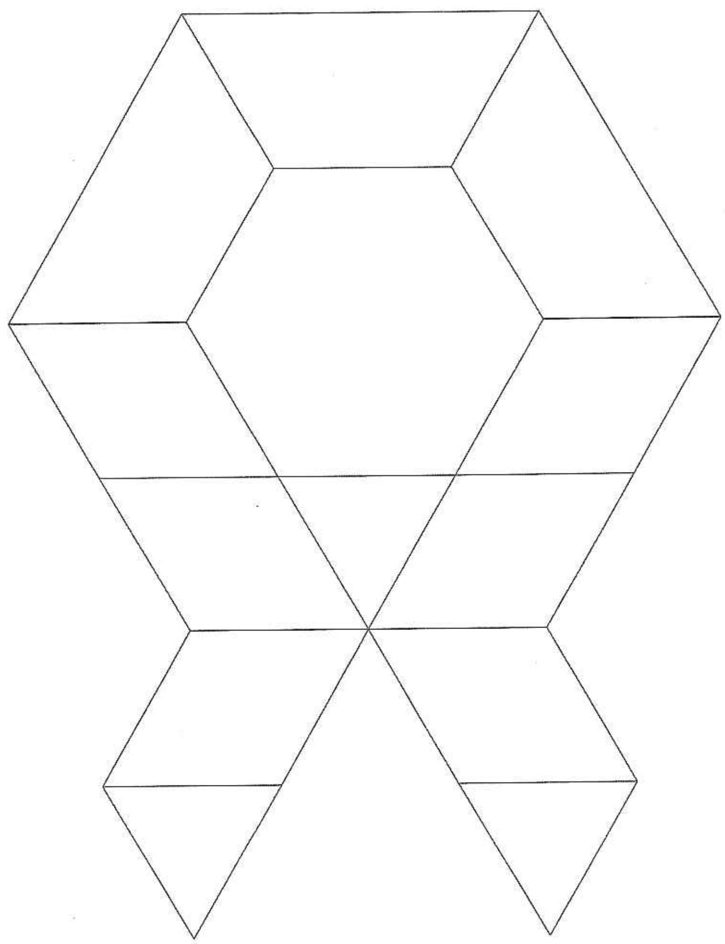
▲ ***Doesn't a class discussion give away the answer before students work on their own?***

If this were a testing situation rather than a learning opportunity, I wouldn't have the students discuss their thinking first. But I think that discussions enhance learning, which is my purpose in giving students this problem to solve. Also, students not only have to find an answer to the problem but also have to explain their reasoning. Talking and listening both help students clarify their thinking and prepare them to write. Even if students use someone else's ideas, they still have to express them in their own words.

▲ ***Why do you have students write their names on the papers they check?***

Having students sign is a way for me to be sure that someone did check each paper. Also, having to sign someone else's paper seems to help students take this part of the assignment seriously. In a way, signing makes them accountable. And for papers with errors, signatures tell me which students I need to talk with about the work.

Pattern Block Design



Pattern Block Triangle Paper